Int. J. Heat Mass Transfer. Vol,20. pp 123–129 Pergamon Press 1977 Printed in Great Britain

NATURAL CONVECTION PATTERNS IN A LONG INCLINED RECTANGULAR BOX HEATED FROM BELOW

PART I. THREE-DIRECTIONAL PHOTOGRAPHY

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(*Received* 30 *October* 1975 *and in revised form* 19 *April* 1976)

Abstract—The transient and steady flow patterns in glycerol in a slightly inclined, long box with a cross-sectional width-to-height ratio of 2.0 were photographed by the method of Krishnamurti. The box was rotated about the long dimension as an axis. The stable mode for very small angles of inclination was found to be a series of roll-cells with their axes running in the upslope. As the angle of inclination was increased to five or six degrees the circulation pattern changed to a series of oblique cells. Above seven degrees a single quasi-two-dimensional cell was observed with flow up the inclined heated plate and down the cooled plate. The rate of heat transfer was also measured. The average Nusselt number at first increased with angle of inclination, then decreased as the oblique cells appeared, and finally increased again as the quasi-two-dimensional cell was formed. Significant hysteresis was not observed in either the circulation pattern or the Nusselt number as the angle of inclination was decreased. The studies were carried out for a Rayleigh number of about 12 000.

NOMENCLATURE

g,	acceleration due to gravity;
H,	height of convection box;
k,	thermal conductivity of fluid;
Nu,	average Nusselt number, $=qH/(T_h-T_c)k$;
q,	net, mean, heat flux density between the plates;
Ra,	Rayleigh number, $=g\beta(T_h-T_c)H^3/\alpha\nu$;
x,	coordinate in the short dimension of the box;
y,	coordinate in the long dimension of the box;
z,	coordinate in the depth direction of the box;
T_h,	temperature of the heated plate;
T_c,	temperature of the cooled plate.

Greek symbols

α,	thermal diffusivity;
β,	volumetric coefficient of expansion with temperature;
θ,	angle of inclination of the box from the horizontal plane;
ν,	kinematic viscosity.

1. INTRODUCTION

NATURAL convection in horizontal and inclined layers and rectangular enclosures of fluid, heated from below and cooled from above, has received extensive study owing to the many important applications, including solar heating. However, the behavior is still not well-defined owing to the large number of influential parameters including the Rayleigh number, the Prandtl number, the two aspect ratios (in the case of a rectangular enclosure), the thermal boundary conditions (including insulated or conducting side walls) and the fluid boundary conditions (including a drag-free upper surface in the horizontal case).

The preferred mode of circulation between infinite horizontal plates is known to be a series of two-dimensional roll-cells. On the other hand, the preferred mode in a horizontal rectangular enclosure has been found to be a series of roll-cells whose axes are parallel to the short side of the box if the aspect ratio (width/height) of the short side is greater than unity [1, 2]. The preferred mode in an inclined enclosure is more difficult to generalize. Hart [3] studied the motion of a shallow layer of water in a differentially heated box and made a diagram of the preferred modes of circulation as a function of Rayleigh number and the angle of inclination. Korpela [4] observed both longitudinal and perpendicular roll-cells in air.

Ozoe *et al.* [5] studied the rate of heat transfer and the mode of circulation in a long, inclined, square channel experimentally and theoretically and found that the minimum in the heat-transfer rate corresponded to the transition from a series of perpendicular roll-cells to a single longitudinal roll-cell. They subsequently [6, 7] reported similar results for rectangular channels and observed that the angle of inclination for the transition in mode of circulation depends strongly on the aspect ratio. They recently presented the results of a three-dimensional analysis of natural convection in several geometries [8, 9] but their results for inclined enclosures are limited to a long channel with a square cross-section.

Holst and Aziz [10], Bories and Combarnous [11] and Weber [12] studied natural convection in sloping porous media. Bories and Combarnous described two types of motion and presented schematic drawings of a longitudinal coil for a series of roll-cells with axes in the upslope. However, their only photographs were for a hexagonal cell.

Krishnamurti [13, 14] developed an ingenious method of photographing from the side the flow pattern in a horizontal plane of fluid. Such a technique is very helpful since the top and bottom surfaces should be very good thermal conductors, and few materials are both transparent and good thermal conductors. The further development and use of this scheme is described herein.

Photographs of particle tracks in three orthogonal planes in the convection cell are presented, together with related measurements of the rate of heat transfer. A three-dimensional theoretical analysis of the fluid motion and rate of heat transfer for this situation will be presented in a subsequent paper. The results herein are limited to a single enclosure and a single rate of heating in glycerol corresponding to Rayleigh numbers of about 12000.

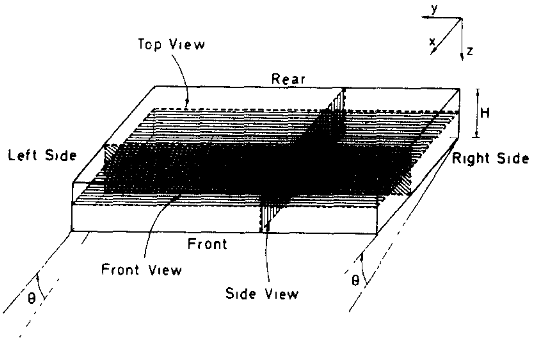


FIG. 2. Configuration of photographic planes.

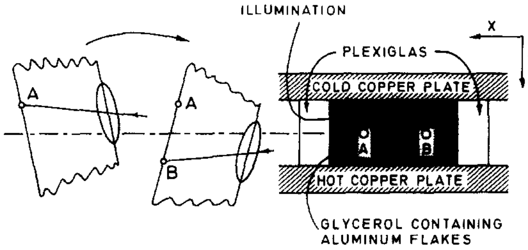


FIG. 3. Camera orientation.

2. EXPERIMENTAL APPARATUS AND TECHNIQUE

Krishnamurti [13, 14] described the essentials of a technique to photograph the flow pattern in a horizontal plane for natural convection between two flat plates. Insufficient details were provided to permit copying this device but the concept was used as a guide to the development of the apparatus and technique utilized herein. Figure 1 is a schematic diagram of the optical source. A collimated beam of light from a laser passes through the fluid in the horizontal direction. Light reflected $\pi/2$ rad by particles suspended in the fluid is registered on the film of a camera. As the light beam traverses the convection cell from the front to the back, in the plane labelled *Top View* in Fig. 2, the camera is moved in the same direction, changing its angle of observation while keeping the beam in focus as shown in Fig. 3. In this way the horizontal plane traversed by the laser beam is scanned by the film, giving the same view as would be seen from above. The camera was a Nikon F-2 Photomic with a Nikkor S.C. auto 1:1.2, 55 mm lens. Closeup lens were added. The film speed was ASA32.

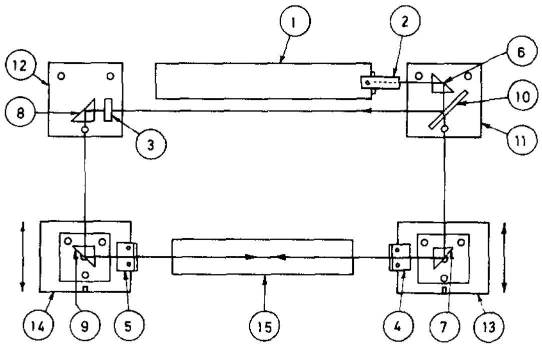


FIG. 1. Schematic diagram of laser light source and convection box. 1—Emitter; 2, 3, 4, 5—Pinholes: 6, 7, 8, 9—Rectangular prisms; 10—Half mirror; 11, 12—Fixed prism stands; 13, 14—Movable prism stands; 15—Convection cell.

The power of the laser was only 5 mW which is not sufficient to heat the particles and the fluid significantly. The laser beam was separated at the exit of the emitter by a half-mirror and the resulting two beams were projected into the convection cell from opposite ends. Considerable care was required to align the two beams along the same path.

The photographic apparatus was mounted on a slide-bearing rail system for good alignment and controlled, smooth motion. The two prisms at the ends of the convection cell were mounted on moving plates on the rails. The camera was fastened to a small box which could be rotated about an axis fixed to a third moving plate on the rails. The angle of the camera body was set so that the width-to-length ratio of the photograph was equal to that of the convection cell. This was accomplished by adjusting the angle of the inclined guide-rail by trial and error.

It was essential to keep the illuminated fluid particles in good focus in order to obtain a clear picture. The refractive index of the glycerol differs from that of air. As the laser beam passes from the front to the rear of the convection cell, the distance travelled by the reflected light varies and the speed of the camera has to be decreased relative to that of the prism holder in the ratio of the indices of refraction of air and glycerol (1:1.47). This was accomplished by adjusting the radius of the guide cylinder for the rail system.

A single motor was used to move the camera holder and the two prism holders. The rotational speed of the motor was controlled by a transformer. A worm gear was used to rotate the guide cylinder very slowly. Long, tangential rods from the guide cylinder were connected

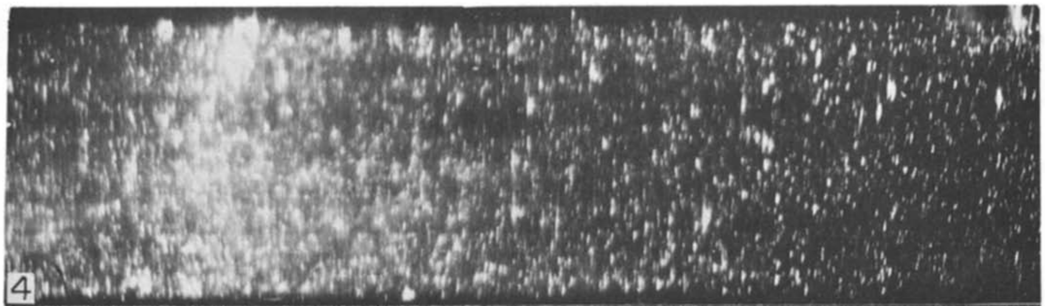
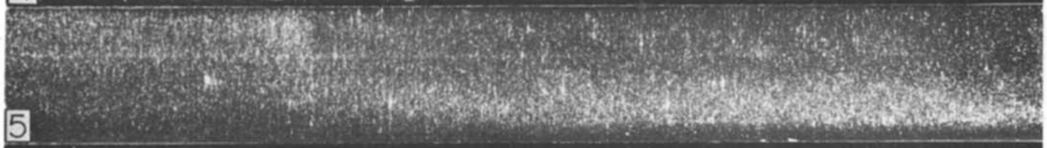
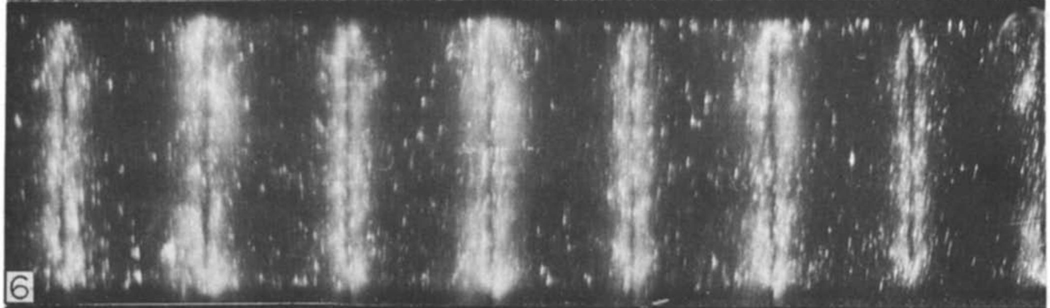
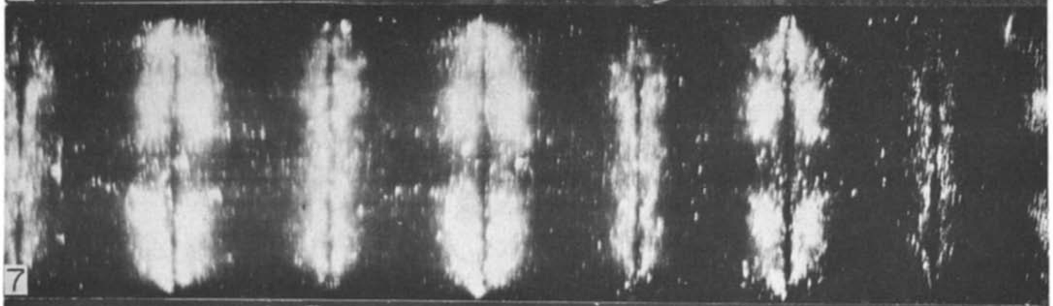
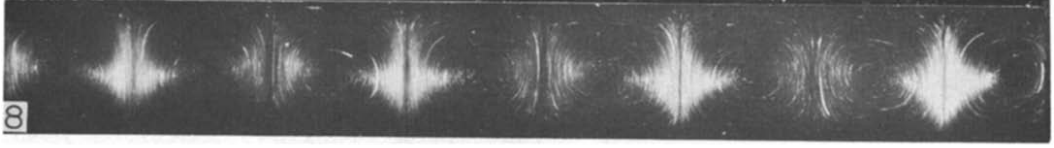






FIG. 4. Top view at 7 mm depth of horizontal box with no heating; f 1.2, 20 s.

FIG. 5. Front view at 20 mm depth of horizontal box with no heating; f 5.6, 60 s.

FIG. 6. Top view at 10 mm depth of horizontal box with heating; f 1.2, 20 s.

FIG. 7. Top view at 7 mm depth of horizontal box with heating; f 1.2, 20 s.

FIG. 8. Front view at 20 mm depth of horizontal box with heating; f 5.6, 20 s.

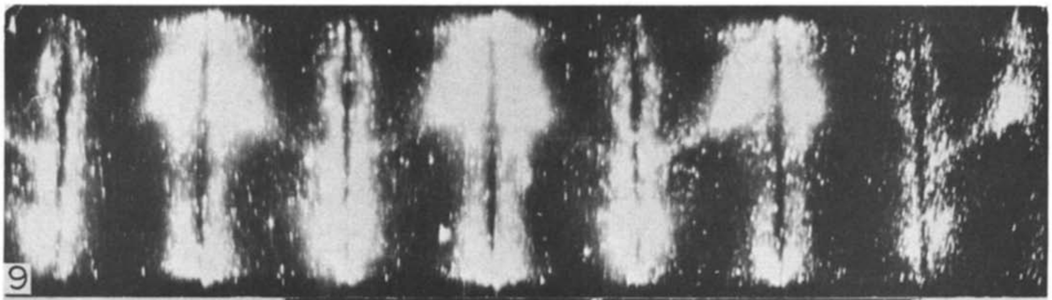
to the moving plates. The traversing speed of the laser beam was 2 mm/s.

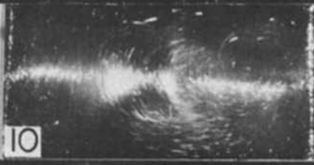
Pictures were taken of the particles in the other two orthogonal planes of the convection cell by projecting a slit of light from a slide projector in the x or y planes and taking a 20–60-s time-exposure at an angle of $\pi/2$ rad.

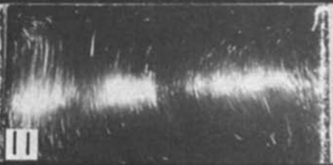
The flow pattern was delineated by a dispersion of aluminum flakes. The aluminum flakes become aligned in the shear flow and hence the amount of reflected light depended on the direction of the shear. The convection cell was 40 × 240 × 20 mm in the x, y and z directions. This geometry was chosen to assure the occurrence of oblique roll-cells.

The whole system, including the photographic apparatus, was mounted on a heavy steel table, and this table was rotated to give different angles of inclination. The error in the measured angle of inclination is believed to be less than 0.01 rad.

The upper and lower surfaces of the convection cell were 8-mm copper plates and the sides were 10-mm Plexiglas. The lower plate was heated electrically and the upper plate was water-cooled. The heat input was measured with a Watt-meter. The net heat input was estimated by subtracting the heat input during a conduction experiment as described in [7] and [15]. The temperature difference between the plates was measured with copper–constantan thermocouples and







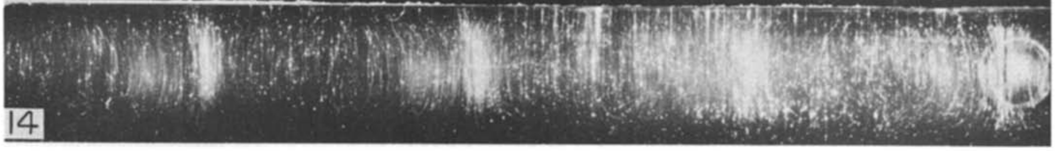


FIG. 9. Top view at 7 mm depth for 0.0524 rad of inclination with heating: *f*/1.2, 20 s.

FIG. 10. Right side view 10 mm from a cell interface for 0.0524 rad of inclination with heating *f*/5.6, 30 s.

FIG. 11. Right side view 5 mm from a cell interface for 0.0524 rad of inclination with heating, *f*/5.6, 30 s.

FIG. 12. Top view at 7 mm depth with heating, 900 s after changing angle of inclination from 0.0524 to 0.0873 rad: *f*/1.2, 20 s.

FIG. 13. Top view at 7 mm depth for 0.0873 rad of inclination with heating, *f*/1.2, 20 s

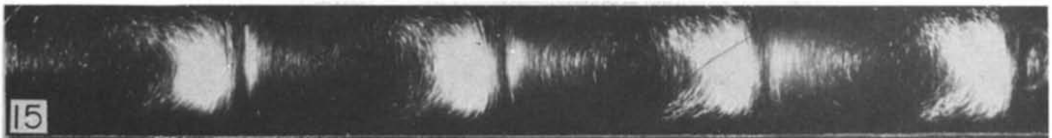
FIG. 14. Front view at 3 mm depth for 0.0873 rad of inclination with heating: *f*/5.6, 60 s.

a precision potentiometer. All runs were made with a heat input of 8 W or none. The variation of temperature over the surface of these plates was estimated to be much less than 0.01 K. The Prandtl number varied from 2720 to 3000.

The sides of the convection cell were covered with glass fiber insulation during the heat-transfer measurements. The insulation was removed to take the photographs. After 600 s of photography the insulation was reinstalled and the apparatus was allowed 1.8×10^4 or more seconds to reach a new thermal steady state.

3. PHOTOGRAPHIC RESULTS

Figures 4 and 5 are for no heating (no motion) and no inclination. Figure 4 is a top view of the right half of the convection box at a depth of 7 mm. The upper side of the photograph corresponds to the back of the enclosure. The brightened area on the left is due to the laser beam from the left. Figure 5 is a front view at a depth of 20 mm with a slit of projector light from the right side only. The laser beam was included in this photograph to indicate the elevation of Fig. 4. These photographs reveal that the aluminum flakes were



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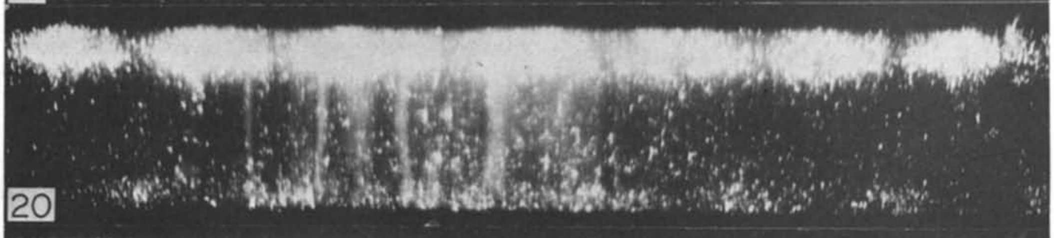

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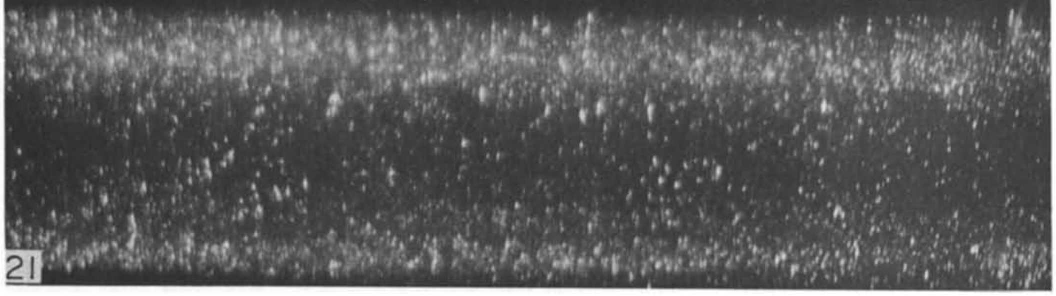

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Fig. 15. Front view at 10 mm depth for 0.0873 rad of inclination with heating; *f*5.6, 30 s.
Fig. 16. Front view at 20 mm depth for 0.0873 rad of inclination with heating, *f*5.6, 30 s.
Fig. 17. Front view at 30 mm depth for 0.0873 rad of inclination with heating, *f* 5.6, 30 s.
Fig. 18. Front view at 37 mm depth for 0.0873 rad of inclination with heating; *f* 5.6, 30 s.
Fig. 19. Top view at 7 mm depth for 0.1047 rad of inclination with heating; *f* 1.2, 20 s.
Fig. 20. Top view at 7 mm depth for 0.1222 rad of inclination with heating; *f* 1.2, 20 s.
Fig. 21. Top view at 7 mm depth for 0.1745 rad of inclination with heating; *f* 1.2, 20 s.

uniformly distributed and oriented, and hence that there was negligible motion.

Figures 6–8 are for heating with no inclination. The 8 W of electrical input resulted in a 9.2 K temperature difference between the plates corresponding to a Rayleigh number of approximately 12 000 and an average Nusselt number of 2.56. Figure 6 is a top view of the right half of the convection box at a depth of 10 mm. The bright areas indicate fluid in vertical motion and the dark lines the interface between the roll-cells. The dark areas at the center of the roll-cells indicate horizontal motion in the core of the cells.

Figure 7 is the same view at a depth of 7 mm. The cells are of alternate brightness, probably because of the difference in direction of rotation and because of the 7-mm rather than 10-mm depth of the photograph. Figure 8 is a front view at a depth of 20 mm and shows the very regular ends of the roll-cells.

Figure 9 is a top view at a depth of 7 mm for 0.0524 rad of inclination. The bright areas extend from the fluid interfaces to the core of the roll-cells at some points, indicating the beginning of the degradation of the normal roll-cells to oblique roll-cells. Figures 10 and 11 are side views at 10 and 5 mm from a fluid interface between the cells. Thus Fig. 10 is near the center of a roll-cell and Fig. 11 near the edge.

Figure 12 is a top view at a depth of 7 mm, taken 900 s after a change in inclination from 0.0524 to 0.0873 rad, with the mode of circulation in process of change from normal to oblique roll-cells. Figure 13 is the same view after the steady state is attained with bell-like shapes appearing alternatively in the front and back of the enclosure. When the laser beam was projected only from the right a series of half-bells was obtained. Figures 14–18 show the front view at depths of 3, 10, 20, 30 and 37 mm with illumination from the right side only. The obliqueness of the roll-cells is very apparent from this series.

Figures 19–21 are top views at a depth of 7 mm for 0.1047, 0.1222 and 0.1745 rad of inclination, respectively. In Fig. 19 the oblique roll-cells are predominant but some dissymmetry relative to Fig. 13 is apparent indicating the imminence of a shift to a single longitudinal cell. In Fig. 20 the motion is almost completely converted to a single longitudinal cell. This cell consists of a flow up the inclined, heated plate and down the cooled upper plate. Some gradation of light along the cell is apparent, indicating residual effects of the oblique cells. In Fig. 21 the transformation is complete.

Decreasing the angle of inclination from 0.1745 to 0 rad in steps produced essentially the same patterns as described above with virtually no hysterisis. The photographs shown above were selected from over 300.

4. HEAT-TRANSFER RESULTS

The experimental heat-transfer data are plotted in Fig. 22. These data were obtained concurrently with the above photographs. An increase in the average Nusselt number can be observed as the angle of inclination increases from 0 to 0.0524 rad corresponding to an increasing rate of circulation of the normal roll-cells. From 0.0524 to 0.1222 rad the average Nusselt number decreases corresponding to the development of oblique cells. Above 0.1222 rad of inclination the average Nusselt number increases again corresponding to the development and increasing rate of circulation of the single longitudinal cell. Hysteresis was negligible as the angle of inclination was decreased back to zero as indicated by the duplicate data points.

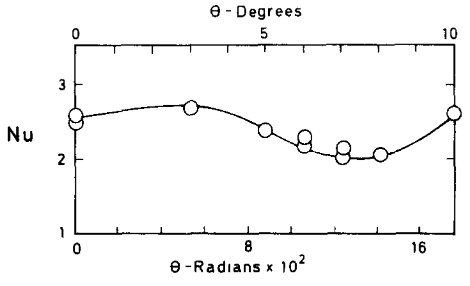

FIG. 22. Average Nusselt number for photographic conditions.

5. CONCLUSIONS

Photographs in three orthogonal planes reveal the motion in an inclined rectangular enclosure heated from below. The photographs in the plane parallel to the heated and cooled plates were taken using the technique of Krishnamurti.

The measurements were confined to a box 20 × 40 × 240 mm, heated and cooled on the 40 × 240 mm surfaces, and rotated about the long dimension as an axis. All measurements were for glycerol at a Rayleigh number of approximately 12 000.

From zero to 0.0524 rad of inclination from the horizontal the stable mode was observed to be a series of roll-cells with their axes in the upslope. Between 0.0524 and 0.1047 rad the roll-cells became oblique. At 0.1222 rad and higher a single cell was observed with flow up the inclined heated surface and down the cooled surface.

The average Nusselt number was found to be closely related to the mode of circulation—increasing from 0 to 0.0524 rad, decreasing from 0.0524 to 0.1222 rad and increasing again above 0.1222 rad of inclination from the horizontal.

Very little hysteresis was observed in either the mode of circulation or the average Nusselt number as the angle of inclination was decreased.

Acknowledgement—We would like to express our sincere appreciation for the efforts of the following students at Okayama University, Merrs. Kazumitsu Yamamoto, Takashi Okamoto and Yujiro Hatta of the Class of 1974 and Messrs. T. Doi, K. Watanabe, T. Ooba and M. Ishimoto of the Class of 1973. The advice of Mr. H. Kamei of the Hitachi Maxwell Co., Ltd., in improving the apparatus is acknowledged with thanks.

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CONFIGURATIONS DE CONVECTION NATURELLE DANS UNE LONGUE CAVITE RECTANGULAIRE INCLINEE ET CHAUFFEE PAR LE DESSOUS—I. PHOTOGRAPHIE TRIDIRECTIONNELLE

Résumé—Les configurations d'écoulements transitoires et établis de glycerol ont été étudiées par photographie dans une longue cavité légèrement inclinée et dont la section présente un rapport largeur–hauteur égal à 2,0 en utilisant la méthode de Krishnamurti. La cavité peut tourner autour de sa longue dimension. Le mode stable pour les angles d'inclinaison très faibles est constitué par une série de tourbillons cellulaires dont l'axe remonte la face inclinée. Lorsque l'angle d'inclinaison est augmenté jusqu'à cinq ou six degrés, le schéma de circulation se change en une série de cellules obliques. Au delà de sept degrés, une seule cellule quasi-bidimensionnelle est observée avec écoulement remontant la plaque chauffée inclinée et descendant la plaque refroidie. Le taux de transfert de chaleur a également été mesuré. Le nombre de Nusselt moyen commence par croître avec l'angle d'inclinaison, puis décroît lorsque les cellules obliques apparaissent et finalement croît à nouveau lorsque la cellule quasi-bidimensionnelle se forme. Aucun phénomène d'hystérésis sensible n'a été observé, lorsque l'angle d'inclinaison a été diminué, ni sur la configuration de l'écoulement, ni sur le nombre de Nusselt. L'étude a été effectuée pour un nombre de Rayleigh d'environ 12000.

NATÜRLICHE KONVEKTIONS- STRUKTUREN IN EINEM LANGEN, GENEIGTEN, RECHTECKIGEN BEHÄLTER, DER VON UNTEN BEHEIZT WIRD—I DREIRICHTUNGS-FOTOGRAFIE

Zusammenfassung—Die instationären und stationären Strömungsstrukturen von Glyzerin in einem leicht geneigten, langen Behälter mit rechteckigem Querschnitt und einem Seitenverhältnis Breite: Höhe von 2:1 wurden mit Hilfe der Methode von Krishnamurti fotografiert. Der Behälter rotierte um die Längsachse. Bei sehr kleinen Neigungswinkeln ergab sich ein stabiles Strömungsbild aus einer Reihe von Rollzellen, deren Achsen mit der Neigungsachse zusammenfielen. Eine Erhöhung des Neigungswinkels auf 5 oder 6° veränderte die Strömungsform in eine Reihe von schrägen Zellen. Bei Neigungswinkeln über 7° wurde eine einzige quasi-zweidimensionale Zelle mit Aufwärtsströmung an der geneigten, beheizten Platte und Abwärtsströmung an der gekühlten Platte beobachtet. Der Wärmeübergang wurde ebenfalls gemessen. Mit wachsendem Neigungswinkel nahm die Nusselt–Zahl zunächst zu; nach Bildung der schrägen Zellen trat eine Abnahme ein. Im Bereich der quasi-zweidimensionalen Zelle nahm die mittlere Nusselt–Zahl erneut mit dem Neigungswinkel zu. Bei Versuchen mit abnehmendem Neigungswinkel wurde weder für die Strömungsform noch für die Nusselt–Zahlen eine besondere Hysterese beobachtet. Die Untersuchung wurde für eine Rayleigh–Zahl von etwa 12000 durchgeführt.

СТРУКТУРЫ СВОБОДНОЙ КОНВЕКЦИИ В ДЛИННОЙ НАКЛОННОЙ ПРЯМОУГОЛЬНОЙ КЮВЕТЕ, НАГРЕВАЕМОЙ СНИЗУ — I. ТРЕХМЕРНАЯ ФОТОСЪЕМКА

Аннотация — С помощью метода Кришнамурти фотографировалось нестационарное и стационарное течение глицероля в слегка наклонной длинной кювете с отношением ширины к высоте, равным 2. Кювета вращалась вокруг своей длинной стороны как вокруг оси. Найдено, что при малых углах наклона устойчивая структура течения представляет собой ряд вихрей в виде ячеек с осями, направленными вверх. С увеличением угла наклона до 5 или 6 градусов картина циркуляции изменялась и представляла собой уже ряд наклонных ячеек. При угле наклона свыше 7 градусов наблюдалась единственная квази-двухмерная ячейка с восходящим потоком на наклонной нагретой пластине или нисходящим на охлажденной пластине. Измерялась также скорость теплопереноса. Среднее число Нуссельта сначала увеличивалось с углом наклона, а затем уменьшалось при появлении наклонных ячеек и в конечном счете вновь увеличивалось при образовании квази-двухмерной ячейки. С уменьшением угла наклона ни в циркуляции жидкости, ни в числе Нуссельта не наблюдалось значительного гистерезиса. Исследования проводились при числах Релея около 12 000.